

Neutrino masses and mixings with an S_3 family permutation symmetry

Francesco Caravaglios and Stefano Morisi

Dipartimento di Fisica, Università di Milano, Via Celoria 16, I-20133 Milano, Italy

and

INFN sezione di Milano

Abstract

Large neutrino mixing angles suggest that the Yukawa sector is invariant under permutations of the fermion families. This S_3 permutation symmetry is broken at a large energy scale but much below the unification scale. Assuming that the lepton mass matrix is approximately diagonal, all neutrino mixing angles naturally come from the breaking of $S_3 \rightarrow S_2$. In the neutrino sector, S_2 remains (approximately) unbroken. As a consequence, we have a large atmospheric neutrino angle and $U_{e3} = 0$. The S_3 symmetry at the unification scale can also explain the large solar mixing angle. We give an explicit expression of the solar mixing angle in terms of the left-handed neutrino masses.

We observe that this family permutation symmetry comes very naturally from a quantized theory of functionals[1], that is an extension of quantum field theory.

1 Introduction

In the Standard Model (SM) Lagrangian, the breaking of the $SU(2) \times U(1)$ gauge symmetry is due to a Higgs scalar doublet. This is the most realistic possibility since triplets or larger representations give unrealistic masses and mixings angles in the electroweak gauge boson sector. In order to give mass to the SM fermions, we introduce Yukawa interactions between fermions and the Higgs doublet.

Neutrinos are very light with respect to all other fermions. The reason is probably that the right-handed neutrino is $SU(2) \times U(1)$ singlet. It takes a Majorana mass that does not break the electroweak gauge symmetry, and so its mass is expected to be of the order of the Grand Unification scale. The left-handed neutrino has a Majorana mass that is proportional to the inverse of the right-handed neutrino mass. Through this seesaw mechanism we understand why neutrinos are so light.

Also neutrino mixings appear very different from quark mixings. The V_{ckm} matrix is close to the identity matrix, mixing angles are not large. The mixing between the heaviest quarks with the lightest is very small. On the contrary, recent neutrino experiments[2-6] have shown that mixings angles between neutrinos with different lepton flavour are very large, approximately maximal. A good candidate for the MNS matrix, (the analogue of the CKM matrix) is

$$O_{\text{MNS}} = \begin{pmatrix} \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (1)$$

Since we measure $\sin^2 \theta_{\text{sol}}$ and $\sin^2 \theta_{\text{atm}}$ the choice of signs in the matrix elements (1) is not unique. The choice in (1) corresponds to neutrino mass eigenstates that are also eigenstates of the permutation symmetry $\nu_\mu \leftrightarrow \nu_\tau$. This S_2 symmetry directly implies $U_{e3} = 0$ and a maximal atmospheric neutrino mixing angle (see the third column in (1)). The importance of discrete symmetries in the description of neutrino masses and mixings has been observed by several authors[7-20]. In this paper we will show that if we embed S_2 into a S_3 symmetry, that is the permutation symmetry of the three fermion families (including the three right-handed neutrinos), and if we add just one (left-handed) Weyl fermion¹ χ_L , that is both a SM singlet and a S_3 singlet, we can predict the MNS matrix in eq.(1). This S_3 family symmetry arises in a very natural way from a quantized theory of functionals[1], that is an extension of quantum field theory. We will discuss a simple model with three right-handed neutrinos, where the S_3 symmetry is spontaneously broken into S_2 . The right-handed neutrino can get a Majorana mass only when the additional $U(1)$ (included in $SO(10)$) is broken. Before the breaking of the S_3 symmetry, χ_L only mixes with the S_3 singlet right-handed neutrino. This introduces a splitting between the S_3 singlet and doublet in the ν_R sector. The solar angle $\sin \theta_{\text{sol}} = 1/\sqrt{3}$ arises when this splitting is large, while the breaking $S_3 \rightarrow S_2$ implies $U_{e3} = 0$ and $\sin \theta_{\text{atm}} = 1/\sqrt{2}$. In the next section we will discuss a generic S_2 symmetric matrix, and in the third section we will discuss the S_3 symmetric case.

2 S_2 symmetry and neutrino mass and mixings

First, we discuss the most general S_2 symmetric real mass matrix. We require that the mass matrix is invariant when we exchange the 2nd with the 3rd row (and column),

$$P M_\nu^R P = M_\nu^R \quad (2)$$

where P is the S_2 permutation matrix

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

The matrix equation (2) is satisfied if and only if

$$M_\nu^R = \begin{pmatrix} a & d & d \\ d & b & c \\ d & c & b \end{pmatrix} \quad (3)$$

where a, b, c, d are four real parameters. Since all parameters are real, we need just an orthogonal matrix O to diagonalize the symmetric matrix M_ν^R , (3). We already know that the vector $v_3 = (0, -1/\sqrt{2}, 1/\sqrt{2})$ is an eigenvector, since it is the only state with odd S_2 parity: $Pv_3 = -v_3$. So, two of the three angles of the orthogonal matrix O are fixed by the S_2 symmetry, while the third angle (the solar neutrino mixing angle) that mixes the two states orthogonal to ν_3 is free. Any matrix in (3) can be diagonalized by the orthogonal matrix

$$O = \begin{pmatrix} -\cos \theta & \sin \theta & 0 \\ \frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \cos \theta & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (4)$$

¹However this additional fermion χ_L is not necessary. It helps in the explanation of the large splitting between S_3 singlet and doublet right-handed neutrinos.

and the angle $\sin \theta$ is obtained by imposing that the matrix $m_\nu = O^t M_\nu^R O$ must be diagonal.

$$m_\nu = O^t \begin{pmatrix} a & d & d \\ d & b & c \\ d & c & b \end{pmatrix} O \quad (5)$$

or more explicitly

$$m_\nu = \begin{pmatrix} z + x \cos 2\theta - y \sin 2\theta & -x \sin 2\theta - y \cos 2\theta & 0 \\ -x \sin 2\theta - y \cos 2\theta & z - x \cos 2\theta + y \sin 2\theta & 0 \\ 0 & 0 & b - c \end{pmatrix} \quad (6)$$

where

$$\begin{aligned} x &= \frac{1}{2}(a - b - c) \\ y &= \sqrt{2}d \\ z &= \frac{1}{2}(a + b + c). \end{aligned} \quad (7)$$

We have to set to zero the off diagonal entries, and from this condition we derive the solar mixing angle

$$\tan 2\theta = -\frac{y}{x} \quad (8)$$

or

$$\sin 2\theta = \frac{y}{\sqrt{x^2 + y^2}} \quad (9)$$

If we put this value of θ in the matrix (4), the matrix (6) is diagonal and the elements in the diagonal give the three masses

$$\begin{aligned} m_1^R &= z - \sqrt{x^2 + y^2} \\ m_2^R &= z + \sqrt{x^2 + y^2} \\ m_3^R &= b - c. \end{aligned} \quad (10)$$

3 S_3 symmetry breaking and neutrino masses and mixings

We consider a S_3 symmetric model with three left-handed neutrinos ν_L^i (i is the family index) and three right-handed neutrinos ν_R^i . We also add a left-handed Weyl fermion χ_L , that is a Standard Model singlet ($SU(3) \times SU(2) \times U(1)$ singlet). It does not carry the family index, i.e. it is a S_3 singlet. The scalar sector includes two standard model singlets for each family, ϕ^i and φ^i , plus the common $SU(2)$ scalar doublet H . We assume that the lepton mass matrix is approximately diagonal, while for the neutrinos we consider a Yukawa interaction of the form²

$$L_{\text{yuk}}^a = g \sum_i \bar{\nu}_L^i \nu_R^i H + \lambda_1 \sum_i \bar{\nu}_R^{c\ i} \nu_R^i \phi^i + \lambda_2 \sum_i \bar{\chi}_L \nu_R^i \varphi^i + M \bar{\chi}_L \chi_L^c + \text{h.c.} \quad (11)$$

we have added the $S_3 \times SU(3) \times SU(2) \times U(1)$ singlet, χ_L^c , because we want an explicitly renormalizable Lagrangian and we want to explain the splitting among the three right-handed neutrinos, but such a singlet is not necessary since one could add such a splitting directly to the Lagrangian (without breaking S_3)

$$L_{\text{yuk}}^b = g \sum_i \bar{\nu}_L^i \nu_R^i H + \sum_i \lambda_1 \bar{\nu}_R^{c\ i} \nu_R^i \phi^i + \lambda_3 \sum_{i,j} \bar{\nu}_R^{c\ i} \nu_R^j \psi + \text{h.c.} \quad (12)$$

²The scalar field φ^i does not break S_3 : ($\langle \varphi^1 \rangle = \langle \varphi^2 \rangle = \langle \varphi^3 \rangle$)

L_{yuk}^b is simpler, while L_{yuk}^a and the S_3 singlet χ_L can probably explain the large splitting among the right-handed neutrinos in terms of additional symmetries of the Lagrangian. In any case, there are two important ingredients that appear to be necessary in realistic examples: there must be an S_3 symmetry, at the energy scale where the heaviest right-handed neutrino acquires a mass³, then S_3 must break at a much lower scale. The breaking $S_3 \rightarrow S_2$ occurs at a much smaller scale than the grand unification scale.

To start we take the Lagrangian (12). ψ and ϕ^i are two scalar fields, with $\langle\psi\rangle \gg \langle\phi^i\rangle$. H is the Higgs $SU(2)$ doublet, it does not carry any family index⁴. To simplify the notation we set $\lambda_3 = 1$, since it is an irrelevant scale factor. The mass matrix for the right-handed neutrinos becomes

$$M_\nu^R = \langle\psi\rangle \begin{pmatrix} \frac{\langle\phi^1\rangle}{\langle\psi\rangle} + 1 & 1 & 1 \\ 1 & \frac{\langle\phi^2\rangle}{\langle\psi\rangle} + 1 & 1 \\ 1 & 1 & \frac{\langle\phi^3\rangle}{\langle\psi\rangle} + 1 \end{pmatrix}. \quad (13)$$

Now we assume $\langle\phi^2\rangle = \langle\phi^3\rangle$ such that the S_2 exchange symmetry remains unbroken, and $\langle\psi\rangle \gg \langle\phi^i\rangle$. The Dirac mass matrix $g \bar{\nu}_L^i \nu_R^i H$ that mixes left-handed neutrinos with their right-handed component is proportional to the identity matrix. Then, due to the seesaw, the left-handed neutrinos get a Majorana mass matrix that is proportional just to the inverse of the matrix M_ν^R . Namely,

$$M_\nu^L = g^2 \langle H \rangle^2 (M_\nu^R)^{-1}. \quad (14)$$

The same orthogonal matrix will diagonalize both M_ν^R and M_ν^L . This means that is sufficient to diagonalize M_ν^R . If $\langle\phi^2\rangle = \langle\phi^3\rangle$, the matrix (13) is S_2 symmetric, and it is equivalent to the (3) with $c = d$. From eqs.(7,8,10) we find a relation that constrains the solar mixing angle to be a function of the right-handed neutrino masses (m_1^R, m_2^R and m_3^R)

$$\sqrt{2} \sin 2\theta_{\text{sol}} - \cos 2\theta_{\text{sol}} = \frac{m_1^R + m_2^R - 2m_3^R}{m_2^R - m_1^R}.$$

We can also express the solar mixing angle in terms of the left-handed neutrino masses⁵

$$\sqrt{2} \sin 2\theta_{\text{sol}} - \cos 2\theta_{\text{sol}} = \frac{m_2^L m_3^L + m_1^L m_3^L - 2m_1^L m_2^L}{m_1^L m_3^L - m_2^L m_3^L} = R \quad (15)$$

or

$$\theta_{\text{sol}} = \frac{1}{2} \arcsin \left(\frac{R}{\sqrt{3}} \right) + \frac{1}{2} \arcsin \left(\frac{1}{\sqrt{3}} \right).$$

We remind that m_3^L is the mass of left-handed neutrino corresponding to the S_2 parity odd state. Note that in the limit $m_2^R \rightarrow \infty$, we have $m_2^L \sim 1/m_2^R \rightarrow 0$, and $\sin \theta_{\text{sol}} = 1/\sqrt{3}$. This limit is achieved when the v.e.v. $\langle\psi\rangle$ is much larger than $\langle\phi^i\rangle$. The first v.e.v $\langle\psi\rangle$ breaks $SO(10)$ and leave S_3 unbroken, while the second one breaks S_3 . This is the S_3 approximately symmetric limit, when the S_3 breaking scale is very small compared to the $U(1)$ breaking scale and unification scale.

³That is probably the scale at which the extra $U(1)$ included in $SO(10)$ is broken.

⁴This is the most realistic situation, taking into account constraints coming from electroweak precision tests of the Standard Model.

⁵The left-handed neutrino mass eigenvalues are proportional to the inverse of the right-handed neutrino masses.

4 Conclusions

We have studied the breaking of an S_3 symmetry, that is the permutation of the three fermion families. This breaking naturally explains the large mixing angles among neutrinos of different flavours. We have concluded that while the unbroken S_2 permutation of the μ neutrino with the τ neutrino predicts both the atmospheric mixing angle and $U_{e3} = 0$, the S_3 symmetry below the scale of the $U(1) \subset SO(10)$ breaking gives a simple mechanism to explain the solar neutrino mixing angle. A very simple Yukawa sector, S_3 symmetric like

$$L_{\text{yuk}}^b = g \sum_i \bar{\nu}_L^i \nu_R^i \langle H \rangle + \lambda_1 \sum_i \bar{\nu}_R^{c\ i} \nu_R^i \langle \phi^i \rangle + \lambda_3 \sum_{i,j} \bar{\nu}_R^{c\ i} \nu_R^j \langle \psi \rangle$$

after the breaking of S_3 into S_2 automatically gives

$$\begin{aligned} U_{e3} &= 0 \\ \sin \theta_{\text{atm}} &= 1/\sqrt{2} \\ \sqrt{2} \sin 2\theta_{\text{sol}} - \cos 2\theta_{\text{sol}} &= (m_2^L m_3^L + m_1^L m_3^L - 2m_1^L m_2^L)/(m_1^L m_3^L - m_2^L m_3^L). \end{aligned}$$

The last condition also gives $\sin \theta_{\text{sol}} = 1/\sqrt{3}$ in the limit $m_2^L = 0$. This limit suggests that the breaking of S_3 occurs at an energy scale much below the unification breaking scale. Finally, we observe that the family permutation symmetry derives from a quantized theory of functionals[1], that is an extension of quantum field theory.

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